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2018 HIGHER SCHOOL CERTIFICATE COURSE MATERIALS

Year 10 Headstart Mathematics

Polynomials

Term 3 – Week 3

Name ……………………………………………………….

Class day and time ……………………………………………

Teacher name ………………………………………………...

Term 3 – Week 3 – Theory

The Remainder Theorem:

The Remainder Theorem states that the remainder R of a polynomial $P(x)$ divided by a linear factor $(x-a)$ is equal to $P(a)$, i.e. $R = P(a)$.

Proof:

By long division, all polynomials when divided by a linear factor $(x-a)$ can be expressed in the form:

$P(x) = (x-a).Q(x) + R(x)$

where $P(x)$ is the dividend,$(x-a)$ is the divisor, $Q(x)$ is the quotient and $R(x)$ is the remainder.

We have seen earlier that the degree of $R(x)$ must be less than the degree of the divisor. Since the divisor is a linear factor then $R(x)$ must be a constant or a zero polynomial. So we can write $R(x)$ as R, where R is a constant.

Hence, $P(x) = (x-a).Q(x) + R$.

By substituting $x = a$, then

$P(a) = (a-a).Q(a) + R$

$P(a) = 0.Q(a) + R$

$\therefore P(a) = R$

Example:

By the remainder theorem, find the remainder when $P(x) = 3x^5 - 2x^3 + x^2 - 15$ is divided by $(2x-3)$.

Solution:

$$P\left(\frac{3}{2}\right) = 3\left(\frac{3}{2}\right)^5 - 2\left(\frac{3}{2}\right)^3 + \left(\frac{3}{2}\right)^2 - 15$$

$$\therefore P\left(\frac{3}{2}\right) = \frac{105}{32}$$

$\therefore$ The remainder is $\frac{105}{32}$.

Example:

Given that when the polynomial $P(x) = 2x^4 - ax^2 + 3x - 10$ is divided by $(2x - 1)$ the remainder is 8, find the value of a.

Solution:

$$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^4 - a\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 10$$

$$P\left(\frac{1}{2}\right) = -\frac{67}{8} - \frac{a}{4}$$

Given $P\left(\frac{1}{2}\right) = 8$, so

$$-\frac{67}{8} - \frac{a}{4} = 8$$

$$-\frac{a}{4} = \frac{131}{8}$$

$$\therefore a = -\frac{131}{2}$$

Example:

When $P(x) = 2x^3 - ax^2 + b$ is divided by $(x - 1)$ and $(x + 3)$, the remainders are 5 and -12 respectively, find the values of a and b.

Solution:

$$P(1) = 2(1)^3 - a(1)^2 + b$$

$$P(1) = 2 - a + b$$

Given $P(1) = 5$, so

$$2 - a + b = 5$$

$$b - a = 3 - - - - (1)$$

$$P(-3) = 2(-3)^3 - a(-3)^2 + b$$

$$P(-3) = -54 - 9a + b$$

Given $P(-3) = -12$, so

$$-54 - 9a + b = -12$$

$$b - 9a = 42 - - - - (2)$$

Solving (1) and (2) simultaneously,

Subtract (2) from (1):

$$8a = -39$$

$$\therefore a = -\frac{39}{8}$$

Substitute $a = -\frac{39}{8}$ into (1):

$$b - \left(-\frac{39}{8}\right) = 3$$

$$b + \frac{39}{8} = 3$$

$$\therefore b = -\frac{15}{8}$$

Example:

Find the values of m and n, if $P(x) = x^3 + mx^2 + x + n$ is divisible by $(x^2 - 5x + 6)$.

Solution:

$x^2 - 5x + 6 = (x-2)(x-3)$

$P(2) = 2^3 + m(2)^2 + 2 + n$

$P(2) = 4m + n + 10$

Since $P(x)$ is divisible by $(x - 2)$, then

$P(2) = 0$

So, $4m + n + 10 = 0$

$$4m + n = -10 - - - - (1)$$

$P(3) = 3^3 + m(3)^2 + 3 + n$

$P(3) = 9m + n + 30$

Since $P(x)$ is divisible by $(x-3)$, then

$P(3)=0$

So, $9m+n+30=0$

$$9m+n=-30----(2)$$

Solving (1) and (2) simultaneously,

Subtract (1) from (2),

$5m=-20$

$\therefore m=-4$

Substitute $m=-4$ into (1):

$4(-4)+n=-10$

$-16+n=-10$

$\therefore n=6$

Example:

(i) Show that $P(x)=2x^3+x^2-15x-18$ is divisible by (x^2-x-6).
(ii) By observing the leading term and the constant term, what is the other linear factor of $P(x)$?
(iii) Hence, solve $P(x)\leq 0$.

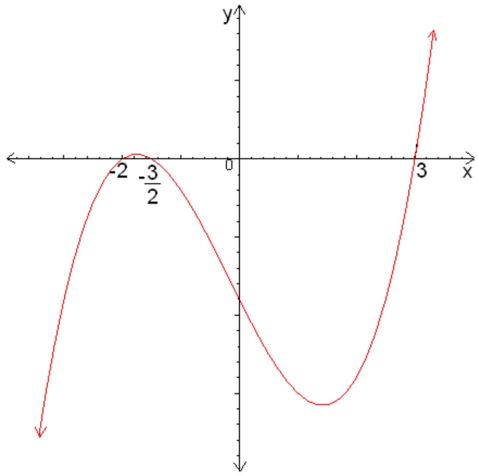
Solution:

(i) $x^2-x-6=(x-3)(x+2)$
$P(3)=2(3)^3+3^2-15(3)-18$
$\therefore P(3)=0$
$P(-2)=2(-2)^3+(-2)^2-15(-2)-18$
$\therefore P(-2)=0$
By the remainder theorem, (x^2-x-6) is a factor of $P(x)$.

(ii) Since the leading term is $2x^3$ and the constant term is -18, then the other factor must be $(2x+3)$.
Test by the remainder theorem,
$$P\left(-\frac{3}{2}\right)=2\left(-\frac{3}{2}\right)^3+\left(-\frac{3}{2}\right)^2-15\left(-\frac{3}{2}\right)-18$$
$\therefore P\left(-\frac{3}{2}\right)=0$
$\therefore (2x+3)$ is the other factor.

(iii) $2x^3 + x^2 - 15x - 18 \leq 0$

$(x-3)(x+2)(2x+3) \leq 0$



$\therefore x \leq -2, \quad -\frac{3}{2} \leq x \leq 3$

The Factor Theorem:

The factor theorem states that if $P(a) = 0$ then $x - a$ is a factor of $P(x)$.

Proof:

By the Remainder Theorem which states that the remainder R of a polynomial $P(x)$ divided by a linear factor $(x - a)$ is equal to $P(a)$, i.e. $R = P(a)$.

In this case, if the remainder is equal to zero when $P(x)$ is divided by $x - a$ then $x - a$ must be a factor of $P(x)$, because $P(x)$ is completely divisible by $x - a$.

The following useful results can be obtained from the Factor Theorem:

For a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$:

(i) If $P(x)$ is a polynomial with integer coefficients and α is an integer zero of $P(x)$, then α is a divisor of the constant term.

(ii) If $P(x)$ has k distinct real zeros $\alpha_1, \alpha_2, \dots, \alpha_k$, then $(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_k)$ is a factor of $P(x)$.

(iii) If $P(x)$ has degree n and n distinct real zeros $\alpha_1, \alpha_2, \dots, \alpha_k$, then $P(x) = a_n(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)$, where a_n is the leading coefficient.

(iv) A polynomial of degree n cannot have more than n distinct real zeros.

(v) A polynomial of degree at most n, which has more than n distinct real zeros, is the zero polynomial (i.e. the polynomial in which $a_0 = a_1 = \cdots = a_n = 0$).

Proof for (i):

If $P(x)$ is a polynomial with integer coefficients and α is an integer zero of $P(x)$, then α is a divisor of the constant term.

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial with integer coefficients, i.e. $a_0, a_1, \ldots, a_n$ are integers.

Suppose α is an integer zero of $P(x)$,

So, $P(\alpha) = 0$ by the Factor Theorem.

$P(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_2\alpha^2 + a_1\alpha + a_0$

$P(\alpha) = \alpha(a_n\alpha^{n-1} + a_{n-1}\alpha^{n-2} + \cdots + a_2\alpha^1 + a_1) + a_0$

$0 = \alpha(a_n\alpha^{n-1} + a_{n-1}\alpha^{n-2} + \cdots + a_2\alpha^1 + a_1) + a_0$

$a_0 = -\alpha(a_n\alpha^{n-1} + a_{n-1}\alpha^{n-2} + \cdots + a_2\alpha^1 + a_1)$

$a_0 = \alpha(-a_n\alpha^{n-1} - a_{n-1}\alpha^{n-2} - \cdots - a_2\alpha^1 - a_1)$

Since a_0 is an integer multiple of α, then

$\therefore$ α is a divisor of a_0.

Example:

(i) Find all the zeros for $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$.
(ii) Hence, factorise $P(x)$ into linear factors.

Solution:

(i) $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$, since all the coefficients are integers then the zeros are the divisors of the constant term.
The divisors of -12 are $\pm1, \pm2, \pm3, \pm4$ and ±6.
$P(1) = 1^4 - 4(1)^3 - (1)^2 + 16(1) - 12 = 0$.
$P(-1) = (-1)^4 - 4(-1)^3 - (-1)^2 + 16(-1) - 12 = -24$.
$P(2) = 2^4 - 4(2)^3 - (2)^2 + 16(2) - 12 = 0$.
$P(-2) = (-2)^4 - 4(-2)^3 - (-2)^2 + 16(-2) - 12 = 0$.
$P(3) = 3^4 - 4(3)^3 - (3)^2 + 16(3) - 12 = 0$.
$\therefore 1, \pm2$ and 3 are the zeros of $P(x)$.

(ii) $\therefore P(x) = (x-1)(x+2)(x-2)(x-3)$

Proof for (ii):

If $P(x)$ has k distinct real zeros $\alpha_1, \alpha_2, \dots, \alpha_k$, then $(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_k)$ is a factor of $P(x)$.

Let $P(x)$ be a polynomial with k distinct real zeros, namely $\alpha_1, \alpha_2, \dots, \alpha_k$.

Then, $P(\alpha_1) = 0$, so $x - \alpha_1$ is a factor of $P(x)$.

Hence, $P(x) = (x-\alpha_1)Q_1(x)$, where $Q_1(x)$ is a polynomial.

Also, $P(\alpha_2) = 0$, so $P(\alpha_2) = (\alpha_2 - \alpha_1)Q_1(\alpha_2) = 0$, but $\alpha_2 - \alpha_1 \neq 0$ so $Q_1(\alpha_2) = 0$.

Thus, $Q_1(x) = (x-\alpha_2)Q_2(x)$, where $Q_2(x)$ is a polynomial.

So, $P(x) = (x-\alpha_1)Q_1(x)$

$P(x) = (x-\alpha_1)(x-\alpha_2)Q_2(x)$

Repeat the same steps k times, it can be shown that $P(x) = (x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_k)Q_k(x)$ for some polynomial $Q_k(x)$.

$\therefore (x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_k)$ is a factor of $P(x)$.

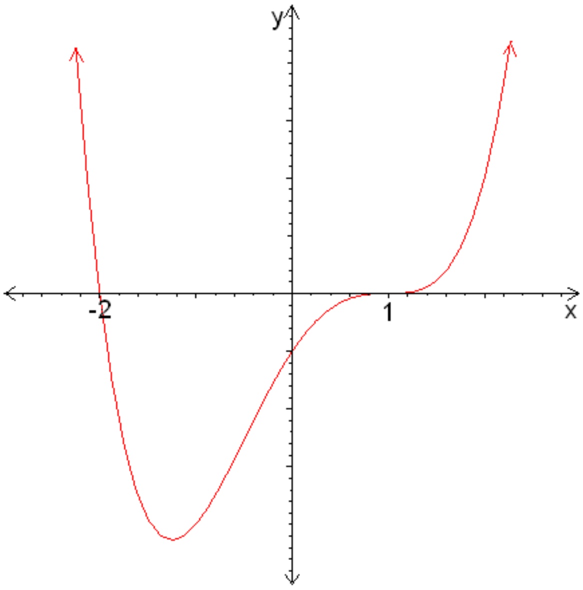
Example:

(i) Factorise $P(x) = x^4 - x^3 - 3x^2 + 5x - 2$ into linear factors.
(ii) Hence, solve $P(x) \leq 0$.

Solution:

(i) $P(x) = x^4 - x^3 - 3x^2 + 5x - 2$, since all the coefficients are integers then the zeros are the divisors of the constant term.
The divisors of -2 are ±1 and ±2.
$P(1) = (1)^4 - (1)^3 - 3(1)^2 + 5(1) - 2 = 0$, so $(x-1)$ is a factor of $P(x)$.
By long division (eliminated), $x^4 - x^3 - 3x^2 + 5x - 2 = (x-1)(x^3 - 3x + 2)$
Let $Q(x) = x^3 - 3x + 2$
$Q(1) = 1^3 - 3(1) + 2 = 0$, so $(x-1)$ is a factor of $Q(x)$
By long division (eliminated), $x^3 - 3x + 2 = (x-1)(x^2 + x - 2)$
So, $P(x) = x^4 - x^3 - 3x^2 + 5x - 2$
$= (x-1)(x^3 - 3x + 2)$
$= (x-1)(x-1)(x^2 + x - 2)$
$= (x-1)^2(x^2 + x - 2)$
$= (x-1)^2(x-1)(x+2)$
$\therefore P(x) = (x-1)^3(x+2)$

(ii) $P(x) \leq 0$
$(x-1)^3(x+2) \leq 0$



$\therefore -2 \leq x \leq 1$

Proof for (iii):

If $P(x)$ has degree n and n distinct real zeros $\alpha_1, \alpha_2, \dots, \alpha_k$, then $P(x) = a_n(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)$, where a_n is the leading coefficient.

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a degree n polynomial with n distinct real zeros $\alpha_1, \alpha_2, \dots, \alpha_n$.

Then by (ii), $(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)$ is a factor of $P(x)$.

So, $P(x) = (x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)Q(x)$, where $Q(x)$ is a polynomial.

Now, both $P(x)$ and $(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)$ are of degrees n, then $Q(x)$ must be a constant.

By comparing coefficients of leading terms x^n,

$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 = (x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)Q(x)$

$Q(x) = a_n$.

$\therefore P(x) = a_n(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)$, where a_n is the leading coefficient.

Example:

Factorise $P(x) = 2x^3 - 4x^2 - 2x + 4$ into linear factors.

Solution:

$P(x) = 2x^3 - 4x^2 - 2x + 4$, since all the coefficients are integers then the zeros are the divisors of the constant term.

The divisors of 4 are $\pm 1, \pm 2$ and ± 4.

$P(1) = 2(1)^3 - 4(1)^2 - 2(1) + 4 = 0$, so $(x - 1)$ is a factor of $P(x)$.

$P(-1) = 2(-1)^3 - 4(-1)^2 - 2(-1) + 4 = 0$, so $(x + 1)$ is a factor of $P(x)$.

$P(2) = 2(2)^3 - 4(2)^2 - 2(2) + 4 = 0$, so $(x - 2)$ is a factor of $P(x)$.

So, $(x - 1)(x + 1)(x - 2)$ are factors of $P(x)$.

Since the leading coefficient of $P(x)$ is 2 then,

$\therefore P(x) = 2(x - 1)(x + 1)(x - 2)$

Proof for (iv):

A polynomial of degree n cannot have more than n distinct real zeros.

This can be proved by **contradiction**.

Suppose a polynomial $P(x)$ of degree n has $n + 1$ distinct real zeros. Then, $P(x)$ has $n + 1$ factors, namely $(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n)(x - \alpha_{n+1})$.

By (iii), $P(x) = a_n(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n)(x - \alpha_{n+1})$ so $P(x)$ is a polynomial of degree $n + 1$.

Hence, this is a contradiction because $P(x)$ is a polynomial of degree n.

$\therefore$ A polynomial of degree n can have at most n distinct real zeros.

Example:

How many distinct real zeros can the polynomial $P(x) = 3x^4 + 4x - 2$ have?

Solution:

The polynomial $P(x) = 3x^4 + 4x - 2$ can have at most 4 distinct real zeros.

Proof for (v):

A polynomial of degree at most n, which has more than n distinct real zeros, is the zero polynomial (i.e. the polynomial in which $a_0 = a_1 = \cdots = a_n = 0$).

This can be proved by **contradiction**.

Suppose a polynomial $P(x)$ had degree and it is at most n.

Also, suppose $P(x)$ has $n + 1$ distinct real zeros. So by (iii), $P(x) = a_n(x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n)(x - \alpha_{n+1})$. Thus, $P(x)$ is a polynomial of degree $n + 1$.

This is a contradiction. $\therefore P(x)$ cannot have a degree.

Since $P(x)$ has no degree then it is the zero polynomial.

Example:

Find the values of p, q and r, if the polynomial $P(x) = 2px^2 + (2q - 3)x + r^2 - 1$ has three zeros.

Solution:

Since the degree of $P(x)$ is 2 then it cannot have more than 2 zeros. It is given that it has three zeros, hence $P(x)$ is the zero polynomial.

So, all the coefficients of $P(x)$ are equal to 0.

$$2p = 0$$
$$\therefore p = 0$$
$$2q - 3 = 0$$
$$2q = 3$$
$$\therefore q = \frac{3}{2}$$
$$r^2 - 1 = 0$$
$$r^2 = 1$$
$$\therefore r = \pm 1$$

$\therefore p = 0, q = \frac{3}{2}$ and $r = \pm 1$.

Example:

(i) The polynomial $P(x) = x^4 + ax^3 - 7x^2 + bx - 18$ has a factor $(x^2 - 3x + 2)$, find the values of a and b.
(ii) Hence, express $P(x)$ in terms of linear factors.

Solution:

(i) Since $(x^2 - 3x + 2) = (x-1)(x-2)$ is a factor of $P(x)$ then,
$P(1) = 1^4 + a(1)^3 - 7(1)^2 + b(1) - 18 = 0$
So, $a + b - 24 = 0$
$a + b = 24 - - - - (1)$
$P(2) = 2^4 + a(2)^3 - 7(2)^2 + b(2) - 18 = 0$
So, $8a + 2b - 30 = 0$
$8a + 2b = 30$
$4a + b = 15 - - - - (2)$
$(2) - (1)$:
$3a = -9$
$\therefore a = -3$
Substitute $a = -3$ into (1),
$-3 + b = 24$
$\therefore b = 27$
$\therefore P(x) = x^4 - 3x^3 - 7x^2 + 27x - 18$

(ii) Given $(x^2 - 3x + 2)$ is a factor of $P(x)$.
By long division (eliminated), $P(x) = (x^2 - 3x + 2)(x^2 - 9)$
$\therefore P(x) = (x-1)(x-2)(x-3)(x+3)$

Example:

When a polynomial $P(x)$ is divided by $(5x-2)(3x+1)$ the remainder is $6x+5$, find the remainder when the polynomial is divided by $(3x+1)$.

Solution:

The polynomial $P(x)$ can be expressed in the following form:

$P(x) = (5x-2)(3x+1)Q(x) + (6x+5)$, where $Q(x)$ is a polynomial.

When $P(x)$ is divided by $(3x+1)$, then

$$P\left(-\frac{1}{3}\right) = \left[5\left(-\frac{1}{3}\right) - 2\right]\left[3\left(-\frac{1}{3}\right) + 1\right]Q\left(-\frac{1}{3}\right) + \left[6\left(-\frac{1}{3}\right) + 5\right]$$

$$= \left(-3\frac{2}{3}\right)(0)Q\left(-\frac{1}{3}\right) + 3$$

$$\therefore P\left(-\frac{1}{3}\right) = 3$$

∴ The remainder when $P(x)$ is divided by $(3x+1)$ is 3.

Term 3 – Week 3 – Homework

The Remainder Theorem:

1. By the remainder theorem, find the remainder when:
   a) $P(x) = x^4 + 2x^2 + 4x - 5$ is divided by x.

   b) $P(x) = x^3 - x^2 + x - 9$ is divided by $(x + 1)$.

   c) $P(x) = x^6 + 2x^5 + 4x - 3$ is divided by $(x - 3)$.

   d) $P(x) = 3x^4 - 8x^2 - 3x + 1$ is divided by $(x - 5)$.

e) $P(x) = 7x - x^2 - 2x^3$ is divided by $(3x + 2)$.

f) $P(x) = 1 - 3x + x^2 - 3x^3$ is divided by $(2x - 1)$.

g) $P(x) = 6x^2 + 5x^3 - 5x^5$ is divided by $\left(x + \frac{1}{3}\right)$.

h) $P(x) = x^8 - 8x^7 + 4x - 4$ is divided by $\left(x - \frac{1}{2}\right)$.

2. When $P(x) = x^4 - x^2 + ax + 1$ is divided by $(x - 3)$, the remainder is 6, find the value of a.

3. When $P(x) = x^3 - 2x^2 + a$ is divided by $(x + 2)$, the remainder is -4, find the value of a.

4. Given that when $P(x) = 2x^3 + 9x^2 + ax + b$ is divided by $(x + 2)$ and $(x - 6)$ the remainders are 1 and 3 respectively, find the values of a and b.

5. Given that when $P(x) = x^4 + 3x^2 - ax + 1$ and $Q(x) = ax^2 - x + 4$ are both divided by $(x - 1)$ the remainders are the same, find the value of a.

6. If $P(x) = x^3 + px^2 - 6x + q$ is divisible by $(x^2 + x - 2)$, find the values of p and q.

7.

(i) Show that $P(x) = x^3 + 5x^2 + 2x - 8$ is divisible by $(x - 1)$ and $(x + 2)$.

(ii) Hence, by observing the leading term and the constant term, express $P(x)$ in terms of linear factors.

8.

(i) Show that $P(x) = x^3 - 3x^2 - 4x + 12$ is divisible by $(x^2 - 4)$.

(ii) Hence, by observing the leading term and the constant term, find the other factor(s) of $P(x)$.

(iii) Solve $P(x) < 0$.

9.

(i) Show that $P(x) = 2x^3 + 5x^2 - 11x + 4$ is divisible by $(x + 4)$ and $(x - 1)$.

(ii) Hence, by observing the leading term and the constant term, express $P(x)$ in terms of linear factors and solve $P(x) \geq 0$.

10. $P(x)$ is a monic polynomial of degree 2. If $(x - 1)$ is a factor and when $P(x)$ is divided by $(x - 2)$ the remainder is 6. Find $P(x)$.

The Factor Theorem:

1. Show that $(x-3)$ is a factor of $P(x) = 9x^3 - 26x^2 - 3x$.

2. Show that $(2x+1)$ is a factor of $Q(x) = 2x^4 + 3x^3 - x^2 - 3x - 1$.

3. Show that $(3x-1)$ is a factor of $P(x) = 6x^3 - 2x^2 + 9x - 3$.

4. Show that $(x-2)$ is a factor of $P(x) = 2x^3 + 6x^2 - 7x - 26$.

5.

(i) Find all linear factors of $P(x) = x^3 - 2x^2 - 5x + 6$.

(ii) Hence, solve $P(x) \leq 0$.

6.

(i) Find all linear factors of $P(x) = 2x^3 + 9x^2 + 3x - 4$.

(ii) Hence, solve $P(x) > 0$.

7.

(i) Show that $(x + 3)$ is the only factor of $P(x) = 2x^3 + 3x^2 - 5x + 12$.

(ii) Hence, sketch $P(x)$.

8.
(i) Show that $(2x+3)$ is the only factor of $P(x) = 10x^3 + 19x^2 + 8x + 3$.
(ii) Hence, sketch $P(x)$.

9. Show that $x^3 - 27$ has only one real root.

10. Find the value of k such that $P(x) = 3x^3 + 2x + k$ is divisible by $(x-1)$.

11. Find the values of a and b such that $P(x) = ax^4 + 2x^3 + bx - 4$ is divisible by both $(x + 1)$ and $(x - 1)$.

12. Given that $(x - 4)$ is a factor of $P(x) = x^3 - 7x^2 + 3x + k$, find the value of k.

13.
(i) If $(x^2 + x - 6)$ is a factor of $P(x) = x^4 + ax^2 + bx + 120$, find the values of a and b.
(ii) Hence, find the other two linear factors.

14.

(i) If $(x^2 + x - 12)$ is a factor of $P(x) = 2x^4 + ax^3 - 17x^2 + bx - 24$, find the values of a and b.

(ii) Hence, find the other two linear factors.

15.

(i) Show that $(x - 3)(x + 2)$ are factors of $P(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$.

(ii) Without long division, find the other factors of $P(x)$.

(iii) Hence, solve $P(x) \leq 0$.

16. Find the values of a, b, c and d, if the polynomial $P(x) = (a^2 - 4)x^3 + (3 - 2c)x^2 + (2b + d)x + b - d$ has five zeros.

17. When a polynomial $P(x)$ is divided by $(3x + 2)(x + 1)$ the remainder is $5x - 6$, find the remainder when the polynomial is divided by $(3x + 2)$.

18. When a polynomial $P(x)$ is divided by $x^2 - 9$ the remainder is $2x^2 + 5$, find the remainder when the polynomial is divided by $(x + 3)$.

19. Write $-3x^2 + 16x - 12$ as a polynomial in terms of $(x - 2)$.

20. Write $2x^3 - 6x^2 + 10x - 1$ as a polynomial in terms of $(x - 1)$.

End of Homework