2018 HIGHER SCHOOL CERTIFICATE
COURSE MATERIALS

HSC Mathematics General II

Loans and Annuities

Term 2 – Week 7

Name ……………………………………………………….

Class day and time ……………………………………….

Teacher name …………….……………………………...

Term 2 – Week 7 – Theory

FLAT RATE LOANS

A flat rate loan is a loan where simple (flat) interest is charged on the principal (amount borrowed). The term of the loan is usually short at around 1-5 years. There may be fees and charges which can be added to the loan or may be paid upfront.

An example of a flat rate loan is personal loans for purchasing goods such as cars or computers.

SIMPLE INTEREST FORMULA

$$I = Prn$$

Where: I = Interest

P = principal (initial amount borrowed)

r = interest rate per period (expressed as a decimal)

n = number of periods

Example 1

Sandra borrowed $20 000 for 5 years at 9% p.a. flat interest to buy a new car.
 a) How much interest does Sandra have to pay on the loan?
 b) How much does Sandra have to repay altogether?
 c) What is Sandra's monthly repayment?

Solution 1
 a) $P = \$20\,000 \quad r = 0.09 \quad n = 5$

$I = Prn$

$I = \$20\,000 \times 0.09 \times 5$

$\quad = \$9000$

Sandra pays $9000 in interest

 b) $Total\ repayment = principal + interest$

$= \$20\,000 + \$9000$

$= \$29\,000$
 c) 5 years = 12 months $\times$ 5 = 60 months

Monthly repayment $= \frac{\$29\,000}{60}$

$= \$483.33$

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Example 2

Katy took out a personal loan of $12 000 over 3 years at 12% $p.a.$ to buy her motorbike. The bank charges were establishment fee $150 and loan insurance $76.50, and Katy added these charges to the principal.
- a) Find her fortnightly repayment
- b) Calculate the interest saved if she had paid the charges upfront

Solution 2

a) Total amount borrowed $= \ \$12\,000 \ + \ \$150 \ + \ \$76.50$

$$= \$12\,226.50$$

Note: interest is charged on the principal and the extra charges as Katy has added them to her loan

$P \ = \ \$12\,226.50 \ r \ = \ 0.12 \ n \ = \ 3$

$I \ = \ Prn$

$I \ = \ \$12\,226.50 \ \times \ 0.12 \ \times \ 3$

$\quad = \ \$4401.54$

Total repayment $=$ Principal + Interest

$$= \$12\,226.50 \ + \ \$4401.54$$

$$= \ \$16\,628.04$$

3 years $= \ 26$ fortnights $\times$ 3 = 78 fortnights

Fortnightly repayment $= \dfrac{16628.04}{78}$

$$= \ \$213.18$$

b) $P \ = \ \$12\,000 \ r \ = \ 0.12 \ \ n \ = \ 3$

$I \ = \ Prn$

$I \ = \ \$12\,000 \ \times \ 0.12 \ \times \ 3$

$\quad = \ \$4320$

Interest saved $= \ \$4401.54 \ - \ \$4320$

$$= \$81.54$$

TERM PAYMENTS

Term payments involve paying off an expensive item over time, generally after a deposit has been made. If the customer fails to pay their regular payments, a higher interest may be charged or the item may be repossessed (taken back).

A term payment plan is sometimes referred to as a hire purchase as the customer is hiring the good before they pay it off completely.

Example 1

A television is advertised for $2000 cash or on terms of 10% deposit and $80 per month for 3 years.
   a)   How much more does the television cost when bought on a term payment plan rather than with cash?
   b)   What is the percentage flat rate of interest charged p.a.?

Solution 1
   a)   Total cost on term payment plan = Deposit + Total payments per month

$$= 0.1 \times \$2000 + \$80 \times 3 \times 12$$
$$= \$3080$$

Extra paid on term payment $= \$3080 - \$2000$

$$= \$1080$$

Note: This represents the interest you have paid
   b)   Balance after 10% deposit $= \$2000 - 0.1 \times \$2000$

$$= \$1800$$

Note: You are finding the amount that you borrowed

$P = \$1800 \quad I = 1080 \quad n = 3$

$I = Prn$

$1080 = 1800 \times r \times 3$

$r = \dfrac{1080}{5400}$

$\quad = 0.2 = 20\%$

Some retail stores may offer a deferred payment plan which means that repayments are not made until a later date, such as after three years. Unpaid balance and outstanding amounts refer to the amount that has been borrowed and has not yet been paid off.

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Example 2

Two competing retail stores offered different deferred payment plans to customers purchasing a refrigerator for $1300 as follows:

Retail A: No deposit needed and no payments for 3 years. After 3 years, flat interest of 2.7% per month charged on unpaid balances from date of purchase

Retail B: No payments required for 300 days. After this period, a flat interest of 0.12%/ day is charged on outstanding amounts from date or purchase.
   a) Which plan should be chosen if a customer wants more time to save up for the repayment of the refrigerator before any interest is charged?
   b) If Amy chooses Retail B's deferred payment plan and paid $950 when the 300 days were up, how much does she owe now?

Solution 2
   a) Retail A's deferred payment plan should be chosen as the deferred payment period of 3 years is longer than Retail B's 300 days.
   b) Unpaid amount $= \$1300 - \$950 = \$350$

   Note: interest charged on $350 and not $1300 because interest is charged on the outstanding balance on date of purchase (Amy already paid $950 before the date of purchase, which is 300 days for Retail B).

   $I = 0.0012 \times \$350 \times 300$

   $\quad = \$126$

   Total owing $= \$350 + \$126$

   $\quad\quad\quad = \$476$

REDUCING BALANCE LOANS

A reducing balance loan only calculates interest on the balance owing (remaining amount of the loan unpaid) and is therefore less expensive than a flat rate loan which charges interest on the principal borrowed. Reducing balance loan takes into account repayments each period which would reduce the balance owing and therefore interest paid.

Some ways to reduce the amount owing and therefore interest paid on a reducing balance loan include:
   • More frequent repayments (e.g. weekly or fortnightly rather than monthly)
   • Making extra payments when you can (e.g. when receiving a bonus or pay rise at work)
   • Increasing the size of repayments (even by a few dollars)

Example 1

Jamie borrowed $600 000 to buy a studio apartment at 7.5% p.a. reducible interest. He made repayments of $5000 monthly. The table below shows the progress of his loan for the first 2 months

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment $(P + I)$	Balance $(P + I - R)$
1	$600 000	$3750	$603 750	$598 750
2	$598 750	$3742.19	$602 492.19	$$597 492.19

a) Show how the interest of $3750 in the first month was calculated.
b) Show the progress of the loan for the next 2 months
c) Find the principal at the start of the $5th$ month
d) How much has Jamie paid off the principal after 4 months
e) How much interest was paid in the first 4 months?
f) How much interest would Jamie pay in 4 months for a loan of $600 000 at a flat interest rate of 7.5% $p.a.$

Solution 1

a) $I = Prn$

$$I = \$600\,000 \times \left(\frac{0.075}{12}\right) \times 1$$

$$= \$3750$$

Note: The interest rate was given in p.a. Therefore, must divide by 12 to calculate rate for the month

b)

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment $(P + I)$	Balance $(P + I - R)$
1	$600 000	$3750	$603 750	$598 750
2	$598 750	$3742.19	$602 492.19	$597 492.19
3	$597 492.19	$3734.33	$601 226.52	$596 226.52
4	$596 226.52	$3726.42	$599 952.94	$594 952.94

c) Principal 5th month = Balance of 4th month = $594 952.94
d) Amount paid off = $600 000 − $594 952.94

$$= \$5047.06$$

e) Interest paid in first 4 months = $3750 + $3742.19 + $3734.33 + $3726.42

$$= \$14\,952.94$$

f) Interest paid on flat rate loan = $600 000 $\times$ 4 $\times \frac{0.08}{12}$

$$= \$16\,000$$

COMPOUNDING INTEREST

Compound interest involves the reinvestment of interest on top of the principal. Therefore, interest will increase as the interest that is reinvested will also earn interest (simple interest only calculated interest on the principal).

COMPOUND INTEREST FORMULA

$$A = P(1 + r)^n$$
$$I = A - P$$

Where:

A = Final amount – also called the future value (FV) or compounded value

P = Principal – also called the present value (PV)

r = interest rate per compounding period expressed as a decimal)

n = number of compounding periods

I = compound interest earned

Example 1

$9000 is invested for 4 years at 7.5% p.a., compounded monthly. Calculate the interest earned.

Solution 1

$PV = \$9000$

12 months in a year, so the rate for one month:

$r = \dfrac{0.075}{12} = 0.00625$

$n = 12\ months\ \times\ 4\ years = 48$

$FV = PV(1 + r)^n$

$\quad = \$9000(1 + 0.00625)^{48}$

$\quad = \$12137.39$

$I = FV - PV$

$\quad = \$12137.39 - \$9000 = \$3137.39$

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Example 2

Calculate the present value (principal) that needs to be invested for 8 years at 8% compounded quarterly to grow to a future value of $12 000

Solution 2

$FV = \$12000$

Since 4 quarters in a year, the quarterly rate:

$r = \dfrac{0.08}{4} = 0.02$

$n = 4 \; quarters \; x \; 8 \; years = 32$

$FV = PV(1 + r)^n$

$\$12\,000 = PV(1 + 0.02)^{32}$

$PV = \dfrac{\$12000}{(1.02)^{32}}$

$= \$6367.60$

CREDIT CARDS

Using a credit card is like taking out a short-term loan as you purchase goods now and pay for them later when you receive a monthly statement. Interest will be charged daily on each item from the date of purchase and may be flat or compound.

Features of credit cards:

- More convenient than carrying cash
- Useful for purchases online, by phone or by post
- Record of spending through monthly statement
- Many credit cards have an interest-free period of 55 days to repay the amount owing in full, otherwise interest is calculated from the date of each purchase
- Interest-free period does not apply to withdrawals (cash advances) made, which will be charged a higher interest rate than purchases.
- A credit limit can be a disadvantage as you are not able to spend as much money as required but can be an advantage because it will prevent you from overspending.

Example 1

Jenny's credit card has a flat interest rate of 12% p.a. and no interest-free period. Her purchases from 1 January to 31 January include:

2 January   Phone recharge $85

23 January   Dinner $87.50

25 January   Jeans $110

If Jenny pays his account in full on $5th$ February, how much does she pay?

Solution 1

The interest for each purchase must be calculated separately. Remember, interest is calculated on each item daily from the date of purchase.

Daily interest rate:

$$r = \frac{0.12}{365}$$

Purchase amount	No. of days' interest (from date of purchase till repayment – $5th$ February	Interest (Note: flat interest rate, use simple interest formula - $I = Prn$)
$85	$31 - 2 + 5 = 34$	$\$85 \times \dfrac{0.12}{365} \times 34 = \$0.95$
$87.50	$31 - 23 + 5 = 13$	$\$87.50 \times \dfrac{0.12}{365} \times 13 = \$0.37$
$110	$31 - 25 + 5 = 11$	$\$110 \times \dfrac{0.12}{365} \times 11 = \$0.40$
Total = $282.50		Total = $1.72

Total purchases $= \$282.50$

Total interest $= \$1.72$

Total payment $= \$282.50 + \$1.72$

$\qquad\qquad\quad = \$284.22$

Example 2

Firstbank

Statement period	14/12/14 to 13/1/15
Account number	2255 4455 8744 4122
Credit limit	$5000
Available credit	$3824.38

Miss Spent

16 Dollar Lane

Decimal Point 2178

Account summary

Balance from previous statement	$816.97
Payment and other credits	$500.00CR
Purchases, cash advances	$791.00
Interest and other charges	$67.65
Closing balance	$1175.62
Minimum balance required	$60
Due date	7 Feb 2015

Your transaction record

Date	Reference	Details	Amount
15/12/14	3146532	Payment – thanks	$500.00CR
15/12/14		Interest	$8.65
18/12/14	4367549	Restaurant spend	$85.60
21/12/14	3020542	ATM withdrawal	$100.00
21/12/14	3020543	ATM fee	$4.00
31/12/14	8746380	Ballet tickets	$265.50
2/1/15		Annual fee	$55.00
7/1/15	2345698	Fabulous Fashions	$256.50
12/1/15	6348921	Sam's Hair & Beauty	$83.40

Annual compound interest rate 22.484%

Daily compound interest rate 0.0616%

a) Show how the closing balance of $1175.62 was calculated
b) What is the credit limit on this card?
c) For this period what was the total amount of:
    i)   Cash advances (withdrawals)
    ii)   Purchases
    iii)   Interest and other charges
d) How much was paid by Miss Spent since the previous statement and on what date
e) When does the interest-free period start and end
f) What interest will be charged if this account is paid in full on 6 February

Solution 2

a) Closing balance $= \$816.97 - \$500 + \$791 + \$67.65$

$$= \$1175.62$$

b) $5000
c) I) Cash advances $= \$100$

ii) Purchases $= \$791 - \$100$

$$= \$691$$

iii) Interest and other charges $= \$67.65$

d) $500 on 15 December 2014
e) From 14 December 2014 to 7 February 2015
f) As 6 February falls within the interest-free period, interest is only charged on the cash advance, from 21 December to 6 February (47 days)
g) Using the compound interest formula:

$P = \$100 \qquad r = 0.000616 \qquad n = 47$

$A = \$100(1 + 0.000616)^{47}$

$\quad = \$102.94$

$\text{Interest} = \$102.94 - \$100$

$$= \$2.94$$

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Term 2 – Week 7 – Homework

1. Mrs Trendy obtained a personal loan of $2 000 to purchase a new flat screen television. The loan is to be repaid over 2 years by equal monthly instalments. The interest is charged at a flat rate of 9.5% p.a. and she pays a loan protection fee of 25 cents per $100 borrowed
   a) Find the total charges that she will pay **[1 mark]**

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   b) Calculate Mrs Trendy's monthly instalment. **[2 marks]**

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2. Furniture priced at $20 000 is purchased. A deposit of 15% is paid. The balance is borrowed using a flat-rate loan at 19% per annum interest, to be repaid in equal monthly instalments over five years. What will be the amount of each monthly instalment? Justify your answer with suitable calculations.
    [4 marks]

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3. Lynne invests $1000 for a term of 15 months. Interest is paid at a flat rate of 3.75% per annum. How much will Lynne's investment be worth at the end of the term? **[1 mark]**

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4. Kimberley has invested $3500. Interest is compounded half-yearly at a rate of 2% per half-year.

Compounded values of $1

Period	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	1.010	1.02	1.03	1.04	1.05	1.06
2	1.020	1.040	1.061	1.082	1.103	1.124
3	1.030	1.061	1.093	1.125	1.158	1.191
4	1.041	1.082	1.126	1.170	1.216	1.262
5	1.051	1.104	1.159	1.217	1.276	1.338
6	1.062	1.126	1.194	1.265	1.340	1.419
7	1.072	1.149	1.230	1.316	1.407	1.504
8	1.083	1.172	1.267	1.369	1.477	1.594

Use the table to calculate the value of her investment at the end of 4 years. **[2 marks]**

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5. Chandra and Sascha plan to have $20 000 in an investment account in 15 years time for their grandchild's university fees. The interest rate for the investment account will be fixed at 3% per annum compounded monthly. Calculate the amount that they will need to deposit into the account now in order to achieve their plan. **[2 marks]**

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6. Heather used her credit card to purchase a plane ticket valued at $1990 on 28 January 2011. She made no other purchases on her credit card account in January. She paid the January account in full on 19 February 2011. The credit card account has no interest free period. Simple interest is charged daily at the rate of 20% per annum, including the date of purchase and the date the account is paid. How much interest did she pay, to the nearest cent? **[2 marks]**

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7. A television was purchased for $2100 on 12 April 2011 using a credit card. Simple interest was charged at a rate of 19.74% per annum for purchases on this credit card. There were no other purchases on this credit card account. There was no interest-free period. The period for which interest was charged included the date of purchase and the date of payment. What amount was paid when the account was paid in full on 20 May 2011? **[2 marks]**

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8. In July, Ms Alott received a statement for her credit card account. The account has no interest free period. Simple interest is calculated and charged to her account on the statement date.

Ms I O Alott			**Dollar Bank**
			Credit card statement
Credit limit: $5000			
Statement date: 23 July 2010			
Previous Balance	Payments	Purchases	Interest charged
$529.46	$529.46	$1721.50	?
Date	Purchases	Amount	Closing Balance
29 June	Hi-Fi	$1721.50	?

Annual percentage rate: 19%

Daily percentage rate: 0.0521%

Note: Interest is charged on amounts from (and including) the date of purchase up to (and including) the statement date.

Minimum payment due: $25 or 5% of closing balance whichever is the larger.

What is the minimum payment due on this account? **[2 marks]**

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9. Georgia is on holidays in Summer Springs and decides to use her credit card for some purchases. Her credit card has an interest free period of 40 days and an interest rate of 11.9% p.a. She makes the following purchases for the period 1 January to 31 January.

5th Jan	Swim wear	$210.75
11th Jan	Cruise	$330.25
18th Jan	Souvenirs	$178.96
26th Jan	Hotel	$810.44

(i)   What is the daily interest rate as a decimal to three significant figures? **[1 mark]**

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(ii)   Georgia pays her account in full on the $21st$ of February. Complete the table below **[2 marks]**

Purchase amount	No. of days interest	Interest to 21st February
$210.75		
$330.25		
$178.96		
$510.44		

(iii)   How much interest did she pay in total for her January purchases? **[2 marks]**

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(iv)   How much did she pay altogether? **[2 marks]**

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End of homework