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2018 HIGHER SCHOOL CERTIFICATE COURSE MATERIALS

Preliminary Mathematics General

Driving Safely

Term 3 – Week 7

Name …………………………………………………………….

Class day and time ……………………………………………

Teacher name ………………………………………………...

Term 3 – Week 7 – Theory

BLOOD ALCOHOL CONTENT

Insurance involves Blood Alcohol Content (BAC) refers to the amount of alcohol present in blood and is given in grams per 100mL. For example a BAC of 0.05 suggests that the individual has 0.05 grams (or 50 milligrams) of alcohol in 100mL of their blood.

Alcohol has many negative effects on drivers, including but not limited to:

- Judgement: decreases the ability to reason and respond
- Concentration: decreasing the ability to focus
- Comprehension: becomes more difficult to understand a situation quickly
- Reaction time: takes longer to respond to the situation

CALCULATING BAC

A “standard drink” is defined to contain 10 grams of alcohol.

The formula that is used to compute a person’s blood alcohol content has four parameters:

- The sex of the person
- Number of drinks consumed
- How quickly the drinks are consumed
- The person’s body mass

BAC CONTENT FORMULA

$BAC_{male} = \frac{10N-7.5H}{6.8M}$ and $BAC_{female} = \frac{10N-7.5H}{5.5M}$

Where

$N =$ number of standard drinks consumed

$H =$ number of hours drinking

$M =$ mass (in kg)

The more mass someone has, the more blood they have and so their BAC will be lower all else being equal. This can also be seen in the formula, as if we keep N and H constant but increase M, since the denominator is getting larger the value for BAC decreases.

Note that females have a lower tolerance than males, so if a male and a female who have the same body mass have the same number of standard drinks within the same time period, the male would have a lower BAC.

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The above formula only applies when $10N > 7.5H$, as BAC must always be greater than or equal to zero.

Example 1

Gaben is 19, weighs 72kg and holds a provisional licence with a zero alcohol limit. He started drinking at 6 p.m and had 10 schooners of full strength beer (15 standard drinks) over 6 hours. He stopped drinking at midnight.

a) What is his BAC, to 2 decimal places at midnight?

b) How long did it take before his BAC was back to zero if his BAC was reduced by 0.02 per hour?

c) At what time could he legally drive his car?

Solution 1

a) First, we note that Gaben is a male so the following formula applies

$$BAC_{male} = \frac{10N - 7.5H}{6.8M}$$

$N = 15$, $M = 72$, $H = 6$

$BAC = \frac{10\times15-7.5\times6}{6.8\times72} = 0.214460784 \ldots$

Therefore, Gaben's BAC is 0.21 to two decimal places at midnight.

b) $\frac{0.214460784\ldots}{0.2} = 1.07230392 \ldots$ hours

$0.07230392 \ldots \times 60 = 4.33823529 \ldots$ minutes

Therefore it takes approximately 1 hour and 4.5 minutes for Gaben's BAC to return to 0

c) From part b, at any time after 1.05am Gaben can legally drive his car as his BAC will be zero.

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Example 2

Who has the lowest BAC?

a) a 72kg male who had 8 standard drinks over 4 hours

b) a 65 kg female who had 9 standard drinks over 5 hours

c) an 82kg female who had 7 standard drinks over 6 hours

d) a 93kg male who had 10 standard drinks over 7 hours

Solution 2

Upon inspection we can see that C has a lower BAC than B because: they have a higher body mass, had less standard drinks and consumed these drinks over a longer time interval (and B and C are both female).

So we only need to substitute A,D and C into the BAC equations as B can be eliminated.

A:

$BAC_{male} = \frac{10\times8-7.5\times4}{6.8\times72} \approx 0.102$

C:

$BAC_{female} = \frac{10\times7-7.5\times6}{5.5\times82} \approx 0.0554$

D:

$BAC_{male} = \frac{10\times10-7.5\times7}{6.8\times93} \approx 0.075$

Therefore, C has the lowest BAC.

GETTING BACK TO A BAC OF ZERO

The liver can reduce the BAC at a rate of 0.015 and 0.02 per hour. Sometimes we can make the simplifying assumption that the BAC reduces at a constant rate when drinks are no longer being consumed.

The initial BAC when drinking has stopped can be divided by the rate at which BAC is reducing to compute the time until BAC reaches zero.

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ACCIDENT STATISTICS

The main causes of driving accidents are:

- Distracted drivers
- Speeding
- Driving under the influence of alcohol
- Reckless, impatient or aggressive driving
- Rain and poor weather conditions

Statistics can be applied to establish trends and represent data about driving accidents and come to conclusions about how to improve road safety. Previous statistics knowledge from this course will be assumed in the examples below.

FATAL ACCIDENTS

In a fatal crash, one or more people may die. Statistics regarding fatal accidents for NSW (2013) are below:

- 42% of all fatal accidents were caused by speeding
- 4.6 road deaths per 100 000 population have occurred
- 88% of drink drivers involved in fatal accidents are male

Example 1:

The following table gives the number of road fatalities in NSW from 2002 to 2011

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
No. fatalities	561	539	510	508	496	435	374	453	405	394

Source: Centre for Road Safety, Transport for NSW

a) For the ten year period, find the:

i) Mean number of fatalities per year
ii) Range
iii) Median
iv) Interquartile range

b) Write a sentence discussing the trends in fatalities from 2002 to 2011. Suggest reasons for the trend.

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Solution 1

a)

i) Mean = Total fatalities / Number of years $= \frac{4675}{10} = 467.5$

ii) First we write the scores in ascending order.
$374, 394, 405, 435, 453, 496, 508, 510, 539, 561$
The range is the highest minus lowest score: $561 - 374 = 187$

iii) Median is between the average of the 5th and 6th score, $\frac{453+496}{2} = 474.5$

iv) Interquartile range = Upper quartile – Lower quartile
= 8th score – 3rd score
$= 510 - 405 = 105$

b) The main trend is a decrease in the number of fatalities. The reason for the decline is most likely used to increasing campaigns for road safety awareness.

FATIGUE-RELATED ACCIDENTS

Fatigue is tiredness resulting from mental or physical exertion. The causes of fatigue are usually long periods of driving without taking rests, or sleep deprivation.

Some statistics for fatigue related accidents (for 2013) include:

- 8.2% of accidents are fatigue related
- 12.9% of accidents in Country regions are fatigue related
- Approximately 22% were aged between 17 and 24 years

Example 2

In one year 440 deaths were attributed to alcohol. 224 of these drivers had a BAC over 0.05 and 273 were aged 15-34.

For road accidents attributed to alcohol:

a) What fraction of drivers who died had a BAC over 0.05?
b) What percentage (to the nearest whole number) of the drivers who died were aged 15-34?
c) What percentage of drivers who died were aged 15-34 had a BAC over 0.05?

Solution 2

a) $\frac{224}{440} = \frac{28}{55}$

b) $\frac{273}{440} = 62.045 \approx 62\%$ (to the nearest whole number)

c) Number of people are both aged 15-34 had a BAC over 0.05 is:
$273 + 224 - 440 = 57$ (this can be seen with a Venn diagram)
$\frac{57}{273} \approx 21\%$
Therefore, 21% of alcohol attributable accidents involving 15-34 year olds had a BAC over 0.05.

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SPEED, DISTANCE AND TIME

Speed limits are imposed to ensure driver and pedestrian safety. Common speed limits in NSW:

- School zone: 40km/h
- Residential street: 50km/h
- City street: 60km/h
- Highway: 100km/h (90km/h for L and P platers)

AVERAGE SPEED

The average speed for a journey is defined by the formula

$$s = \frac{d}{t}$$

d = distance travelled for the journey

t = time taken for the journey

s = speed

Ensure that the units are consistent with each other for this formula. For example, if the units for distance are given in kilometres (km), and time in hours (h), then the speed should be given as kilometres per hour (km/h). If the distance is instead given in metres (m), and the time in seconds (s), the speed must be given as metres per second (m/s). This unit for speed is commonly used in physics and engineering.

Take care to note that the average speed is different from instantaneous speed (which is what the odometer on cars are designed to show).

Example 1

Jora left home to travel at 6.00am from Karth to Summerfell a distance of 1027km. He stopped for meal breaks and fuel arrived in Summerfell at 8.30pm.

a) What is his average speed for the trip, correct to the nearest km/h?
b) How far, to the nearest km, could he travel at this speed in 50 minutes?
c) How long, in hours and minutes, would it take to drive 864km at this speed?

Solution 1

a) $d = 1027$km; $t = 14.5$ h (there are 14.5 h between 6.00am and 8.30pm)

$$\begin{aligned} s &= \frac{d}{t} \\ &= \frac{1027}{14.5} \\ &= 70.8275\ldots \\ &\approx 71 \end{aligned}$$

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Therfore, Jora averages a speed of 71km/h

b) 50 minutes $= \frac{5}{6}$ h
$d = s \times t$
$= \frac{5}{6} \times \frac{1027}{14.5}$, using the exact value for speed to prevent rounding errors
$\approx 59.02229885 \ldots$
≈ 59
Therefore, Jora can travel 59km if he maintains this average speed over 50 minutes.

c) $t = \frac{d}{s}$
$t = 864 \div \left(\frac{1027}{14.5}\right)$
$= 12.1986368 \ldots$
Now, $0.1986368 \times 60 = 12$ minutes.
Therefore, it takes 12 hours and 12 minutes approximately to travel 864km if this speed is maintained.

Example 2

Brienne's trip took 9h and 45 minutes at an average speed of 9.5m/s. What is her:

a) Her average speed for the trip, correct to the nearest km/h?
b) Distance travelled, correct to the nearest m?

Solution 2

a) $\frac{9.5m}{s} \times \frac{1km}{1000m} \times \frac{60 \times 60s}{1h} = \frac{9.5 \times 3.6\ km}{1h} = 34.2\ km/h$. (We convert the units of speed from m/s to km/h)

Therefore, Brienne averages a speed of 34 km/h.

b) $d = st$
$s = 34.2$ km/h, $\quad t = 9.75$h
$d \ = 34.2 \ \times 9.75$
$= 333.45$ km
$= 333\ 450$m. Therefore, the distance travelled is 335 450m to the nearest metre.

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STOPPING DISTANCE

There are several factors that need to be considered when determining the stopping distance, that is,the distance travelled by the vehicle between the locations we first sense the hazard until we stop. Firstly, we shall define the terms below:

- Reaction time: the time elapsed between sensing the hazard and actually applying the brakes. Reaction time varies from person to person and also depends on things like fatigue and presence of distractions.

- Reaction distance: the total distance travelled during the reaction time.

- Braking distance: the distance travelled from when the brakes are applied until the car has completely stopped. This is affected by the road surface type, weather conditions, weight of the vehicle, the condition of the brakes and tyres and most importantly, the speed at which the vehicle is travelling when the brakes are applied.

- Stopping distance: the sum of the reaction distance and braking distance.

We can use the speed-distance formula in the previous section to relate the reaction time to the reaction distance. In order to do so, we assume that the speed remains constant (and does not vary too much from the average speed) during the reaction time. This is a reasonable assumption because the brakes haven't been applied yet and the reaction time is usually less than 2 seconds, so the speed won't change much in this small time interval. The formula is repeated below:

$t_{reaction} \times s = d_{reaction}$

Where:

$t_{reaction}$ = reaction time (seconds)

s = speed (metres per second – before brakes applied)

$d_{reaction}$= reaction distance (metres)

We have chosen to use the units above because they are convenient for reaction distances.

Note that the above formula is not present on the formula sheet in this rearranged form. It is only printed on the sheet as presented in the previous section "Average Speed".

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Example 1

A car cruising at 80 km/h travelled 41.8m on an icy road during the time it took for the driver to see a stop sign and start braking. It then travelled 57.2m under brakes before it stopped.

a) What was the driver's reaction time correct to two decimal places?
b) What was the stopping distance?

Solution 1

a) $\frac{80km}{h} = \frac{80\ 000m}{h}$ (converting to metres per hour)

$= \frac{80\ 000m}{h} \times \frac{1h}{3600s}$ (converting to metres per second)

$\approx \frac{\frac{200}{9}m}{s}$, giving a speed of $\frac{200}{9}$ metres per second

$\frac{d}{s} = t$

$d = 41.8\text{m}, s = \frac{200}{9}$ m/s

$t = 41.8 \div \frac{200}{9}$

$= 1.881$ seconds

Therefore, the driver's reaction time, to two decimal places, is 1.88 s.

b) As defined, the stopping distance is the sum of the braking and reaction distance.

$d_{stopping} = 41.8 + 57.2$, as given

$= 99$

Therefore, the stopping distance is 99m.

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BRAKING DISTANCE FORMULA

The next component of stopping distance that we must analyse is the braking distance, $d_{braking}$.

$$d_{braking} = kv^2$$

Where

$d_{braking}$ = braking distance

$k =$ a constant of proportionality that is determined by road surface conditions, wind, and other factors

v = the speed (or velocity) of the vehicle

Note that it does not matter what units we use for velocity and distance. We can choose a value and units for the constant k in such a way that it gives us the correct value for $d_{braking}$. However after we have chosen some value for k based on the given information, we have to be consistent with the units we use in the formula from then on. This is illustrated in the example to follow.

A significant result from this formula is that the braking distance is proportional to the square of the speed the car is travelling at. This means that a car travelling at 40km/h will have four times the braking distance of the same car travelling at 20km/h, and sixteen times (4^2) the braking distance of the same car travelling at 10km/h, all else being equal.

Example 2

A car travelled 84.3m under brakes when initially travelling at 106.3km/h. If $d = kv^2$, find:

a) The constant k correct to 3 significant figures

b) The braking distance, correct to 1 decimal place, for a car applying brakes from a speed of 110km/h

c) the stopping distance, correct to 1 decimal place, of a car travelling at 110km/h if the driver's reaction time is 2 seconds.

Solution 2

a) We choose that k is a constant valid for situations where the units of d is given in metres and the units for v is given in km/h.

$84.3 = k(106.3)^2$

$k = \frac{84.3}{(106.3)^2}$

$= 7.46 \times 10^{-3}$m.km^{-2}h^{-2} (the units for k are selected in such a way that the units for LHS and RHS both match each other)

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b) $d = kv^2$ using value of k from part a,

$d = (7.46 \times 10^{-3}) \times (110)^2$ m

$=$ 90.3m (to 1 d.p)

Therefore, the braking distance is 90.3m

c) Firstly, we must find the reaction distance.

110km/h $=$ 110 000 m/h (converting to m/h)

$= \frac{110000}{3600}$m/s (converting to m/s)

$d_{reaction} = s \times t_{reaction}$

$= \frac{110000}{3600}$m/s $\times$ 2s

$= 61\frac{1}{9}$ m

Since the stopping distance is defined as the sum of the braking and reaction distance, add part c and b answers to get:

$d_{stop} = 61\frac{1}{9} + 90.3 \approx 151.4$ m (to 1 d.p)

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Term 3 – Week 7 – Homework

1. A car travels 441km on a journey and covers this distance in 5 hours and 15 minutes. Calculate the average speed per hour. **[1 mark]**

2. The speed of sound is approximately 335m/s
   i) How far does sound travel in one minute? **[1 mark]**

   ii) Convert your answer to scientific notation. **[1 mark]**

   iii) Convert the speed of sound to km/h. **[1 mark]**

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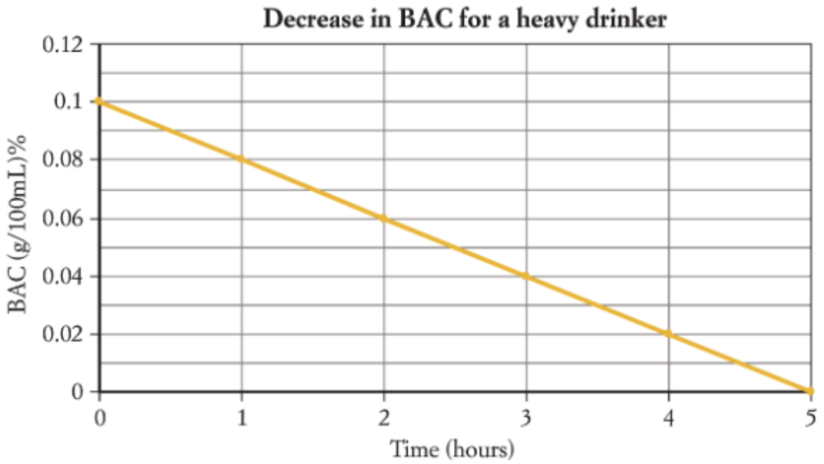
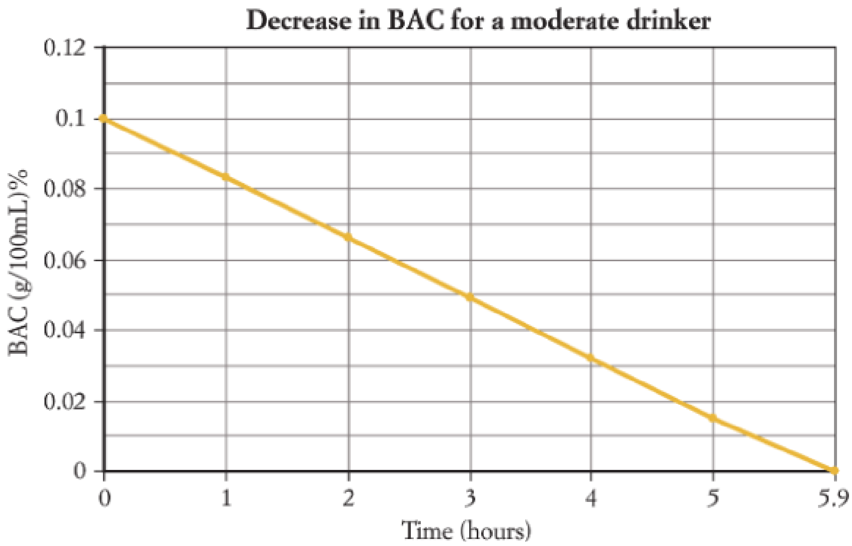
3. If A heavy drinker consumes a large quantity of alcohol most days of the week whereas a moderate drinker consumes less alcohol less frequently. The two graphs show the difference in the decrease of the BAC for a heavy drinker and a moderate drinker over time.





a) Which drinker's BAC returns to zero quicker? **[1 mark]**

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b) What is the rate of decrease (correct to three decimal places) of BAC for: i) a heavy drinker and ii) a moderate drinker **[2 marks]**

c) What is the BAC of a heavy drinker after 1h 30min? **[1 mark]**

d) What is the BAC of the moderate drinker after 4h and 15min? **[1 mark]**

e) What is the difference of the BAC for a heavy and a moderate drinker after 3 hours? **[1 mark]**

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4. A survey revealed that for the fatigued drivers in 18 764 single vehicle crashes, 70.2% were driving cars, 75.5% were male, 35% were between 17 and 24 years and 9.6% were over 60 years old.

a) How many drivers were aged between 24 to 60 years **[2 marks]**

b) How many crashes did not involve cars? **[1 mark]**

c) How many drivers were female? **[1 mark]**

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5. This table shows the categories of people killed in road accidents in 2011.

	NSW	Australia
Drivers	190	579
Passengers	73	286
Pedestrians	52	189
Motorcycle riders and passengers	52	201
Cyclists	10	35
All road users	**377**	**1291**

Source: BITRE

a) What percentage of Australian road deaths occurred in NSW? Answer to 3 significant figures. **[1 mark]**

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b) What percentage, to two decimal places, of road deaths in NSW were pedestrians? **[1 mark]**

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c) How many road deaths in Australia were not drivers or passengers? **[1 mark]**

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What percentage of NSW road fatalities were cyclists? Answer correct to one decimal place. **[1 mark]**

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6. Jax drove his motorbike 460m in 6.011 seconds
   a) What was his speed in m/s correct to one decimal place? **[1 mark]**

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   b) How long, in minutes to 1 decimal place, would it take to go 58km at this rate?**[1 mark]**

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7. A driver has a reaction time of 1.6 seconds. When travelling at 80km/h, he applied his brakes and travelled 62m before stopping. Use the braking distance formula $d = kv^2$ to find:

   a) The constant k correct to 4 decimal places **[2 marks]**

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b) The braking distance, to the nearest 0.1m when travelling at 105km/h **[2 marks]**

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c) The speed, to the nearest km/h, of the car that travelled 75m under brakes before stopping **[2 marks]**

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d) The stopping distance, from a speed of i) 80km/h ii) 105km/ **[2 marks]**

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8. Alex had a BAC of 0.06 when he stopped drinking and it took 5 hours for his BAC to return to zero. At what rate per hour was his BAC reducing ? **[1 mark]]**

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9. Mary is 17, weighs 61kg and holds a provisional licence with a zero alcohol limit. She started drinking at 10p.m and had 9 standard drinks over 4 hours.
   a) What was her BAC, correct to 2 decimal places, when she stopped drinking at 2am? [**1 mark]**

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   b) How long did it take before her BAC was back to zero if it was reduced by 0.015 per hour? [**1 mark]**

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   c) At what time could she legally driver her car? [**1 mark]**

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End of homework

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